Theory of Games

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A Game

A game is the set of strategies played by the players in it. Every game has an outcome. The outcome of the game is dependent on the strategy and the counterstrategy played by the opponents.

There are various types of games in Game Theory.

1. Simultaneous games
2. Repeated games
3. Sequential games
4. The game of entry deterrence

The outcome of the game for each strategy played by an agent and the counter-strategy played by his/her opponent is called the payoffs of the game. Payoff can be either positive or negative.

Strategy

- A strategy is a course of action in a game that is played by an agent or player in expectation of an outcome. Any strategy played by a player has an outcome given the actions of the opponent (!). The outcome of a strategy in a game is the expected payoff that each player gets. The payoff of a strategy is either positive or negative.
- A strategy can be a dominant strategy or equilibrium strategy or both.

Payoff

- The expected outcome of a strategy of a game is expressed in terms of gain or loss.
- Payoff is the expected gain or loss that accrues to each player due to a move by the opponent.
- When the expected payoffs corresponding to each player for various strategies played by the players are represented in terms of rows and column entries it is called a ‘Payoff Matrix’.
- In a ‘Payoff Matrix’, each entry corresponds to the payoffs to both parties in the game.

Payoff Matrix of a Game

Strategic actions can involve many players with infinite strategies. For the sake of understanding we limit our analysis involving only two players with finite number of strategies, say two.

The payoff matrix of a game simply depicts the payoff to each player for each combination of strategies that are chosen by both. Payoff means the gain or loss that a player gets when he plays a strategy. Payoff matrix represents the expected payoffs to each participant for each combination of strategies chosen by them.

Equilibrium Strategy

It refers to that strategy that each player chooses to play that gives him best payoff regardless of the strategy chosen by another. This is also called the dominant strategy. Thus, dominant strategy is defined as the optimal strategy that a player chooses to play no matter what the other player does. Whenever there is a dominant strategy of each player in a game, we would predict that it would be the equilibrium outcome of the game.

Nash Equilibrium

Let there be only two players, A and B and the strategies they play are 'YES' and 'NO'.

A's optimal choice depends on what he thinks B will do.

Dominant strategy equilibrium is established when the choice of strategy by a player is optimal or best for all choices by the other player.

But, if both players are intelligent and well-informed , i.e., they are rational, both will want to choose optimal strategies. It must also be kept in mind that what is optimal for a player depends on what choice another player does. Nash equilibrium refers to the pair of strategies that is optimal to a player given the choice by another player.

Nash Equilibrium *Contd.*

Example: ***If A's choice is optimal (means gives maximum payoff) given B's choice.***

If B chooses a strategy and given the choice by B, the best strategy or the optimal strategy played by A shall form the part of Nash Equilibrium.

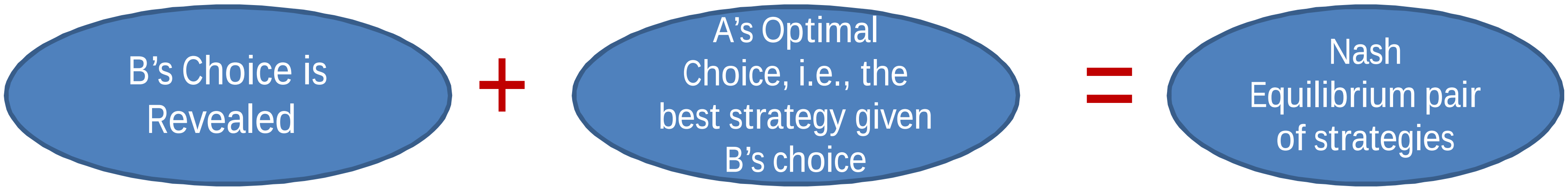
Nash equilibrium is established when the choice of a player is optimal for a given choice by another player.

No player knows what the other player shall do, when he makes decision regarding the choice of strategy. However, he has some expectations about the choice of the other player. Expectations is the average of the probabilities, i.e., what might the other player do on an average if he is rational.

Nash Equilibrium *Contd.*

A Nash equilibrium is a pair of expected payoffs to each player when each of the players have some expectations about the behaviour of the other player, while making his own choice.

*If a person's choice is **revealed**, what would be the other person's optimal choice.*



(Choice by B)

(Choice by A given B's choice)

(Nash Equilibrium pair)

A Game may have more than one Nash Equilibrium pairs

OR

No Nash Equilibrium

Game where there are two Nash Equilibrium pairs:

Case I: When the Structure of the game is *Symmetric*

		Player B	
		YES	NO
Player A	YES	5,2	(0,0)
	NO	(0,0)	2,5

CASE II:Game where there are NO Nash Equilibrium pairs:

There are games that do not have a Nash Equilibrium.
This happens in case of PURE STRATEGIES.

		Player B	
Player A		YES	NO
	YES	0,0	(0,-1)
	NO	(1,0)	-1,3

Nash equilibrium of a game does not necessarily lead to Pareto efficient outcomes (Cournot's Duopoly Model)

		Player B	
		YES	NO
Player A	YES	-3,-3	(0,-6)
	NO	(-6,0)	-1,-1

Nash Equilibrium

(***Generalisations of the Cournot's Equilibrium***)

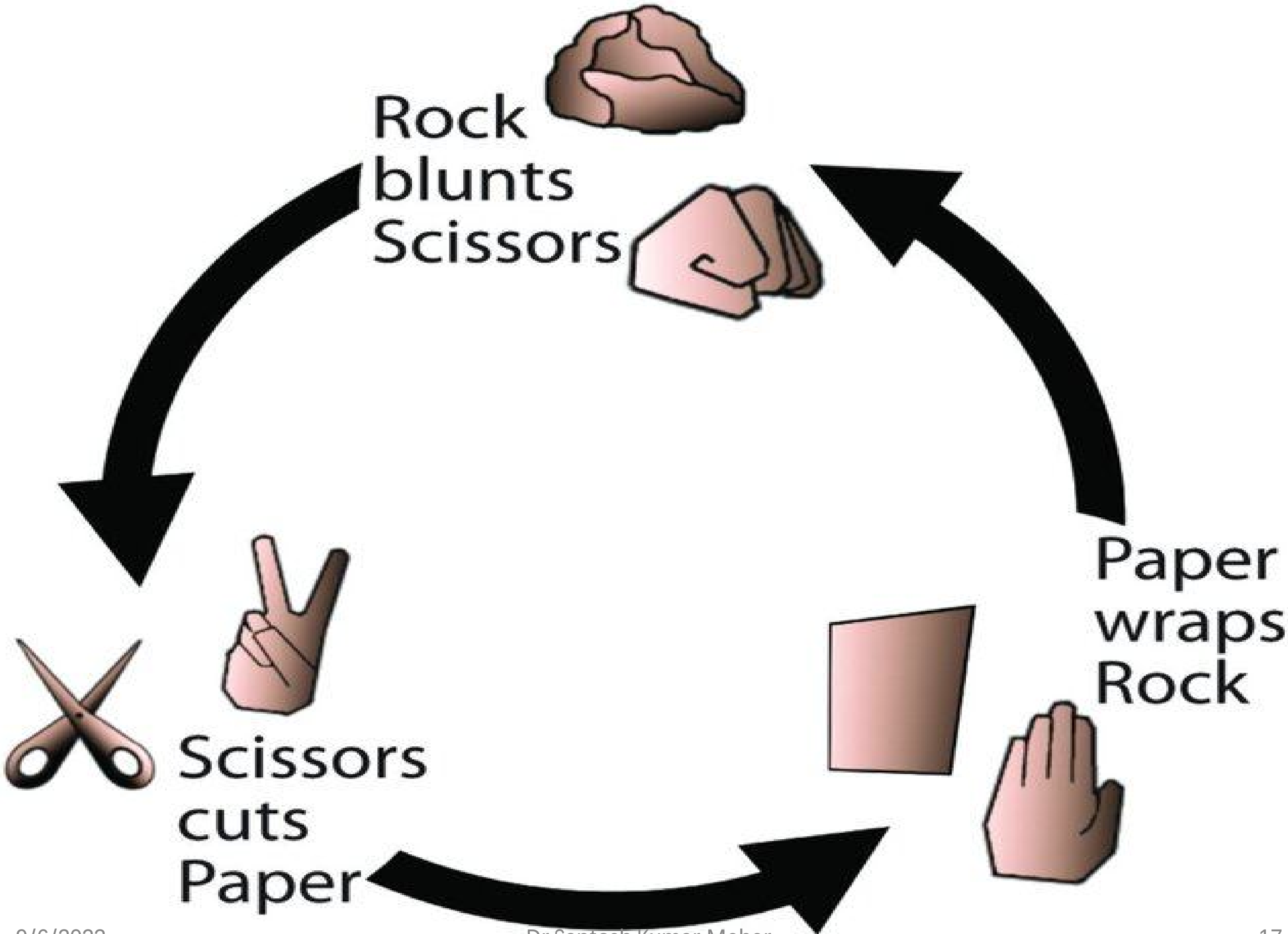
- In Cournot's equilibrium, the choices are the output levels wherein each firm chooses its output level taking the other firm's output decision as fixed (naïve behaviour). Each firm is supposed to do the best for himself assuming a certain behaviour/reaction pattern of the rival firm.
- A Cournot Equilibrium occurs when each firm is maximising profits (optimal choice), given the other firm's behaviour. *This is precisely the definition of Nash Equilibrium.*

Mixed Strategies

- When a player chooses a strategy and sticks to it, it is called a **pure strategy** (Cournot's case). This is a strategy chosen once and for all.
- **Mixed Strategy** is the strategy that includes a mix of strategies chosen by a player. Sometimes players randomize their strategies. That means they assign a probability to each choice and play their choices according to their probabilities.
- ***(For example: A may choose 'YES' 50% of times and 'NO' 50% of times. Similar is the case of B.)***
- ***A Nash equilibrium in mixed strategies refers to an equilibrium in which each player chooses the optimal frequency with which to play his strategies given the frequency choices of the other player.***

Rock Paper Scissors Game

- Each player simultaneously chooses to display a fist (Rock), a palm (Paper) or his first two fingers (Scissors).
- The Rules: Rock breaks Scissors, Scissors cuts Paper, Paper wraps Rock


Rock
blunts
Scissors
Paper
wraps
Rock
Scissors
cuts
Paper

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Rock Paper Scissors Game

Game theorists realise that the equilibrium strategy in Rock Paper Scissors game is to choose one of the three outcomes randomly. But, human minds are not so perfect in choosing totally random outcomes. Thus, if a player can predict the opponent's choice to some degree, he can have an edge over the rival in making his choices.

The ‘Prisoner’s Dilemma’

- One of the problems of the Nash equilibrium is that it does not necessarily lead to Pareto efficient outcomes. Pareto efficiency refers to that equilibrium where one can not be made better off without making others worse off.
- Nash equilibrium is depicted through ‘Prisoner’s Dilemma’ too.

The 'Prisoner's Dilemma'

Nash equilibrium of a game does not necessarily lead to Pareto efficient outcomes (Cournot's Duopoly Model)

		Suspect B	
		Confess	Doesn't Confess
Suspect A	Confess	10,10	(0,20)
	Doesn't Confess	(20,0)	5,5

The ‘Prisoner’s Dilemma’

- ‘Prisoner’s Dilemma’ reveals a paradox in decision making behaviour, in which two prisoners, acting in their self-interests or acting rationally, do not produce Pareto optimal outcome.
- It was framed originally by Merrill Flood and Melvin Dresher in 1950 when they were working at RAND. Albert W. Tucker later formalised the game by structuring the rewards in terms of prison sentences and named it the ‘Prisoner’s Dilemma’.

The 'Prisoner's Dilemma'

•'The prisoners could have gained from cooperation but suffer because they fail to so as they faced difficulties to coordinate their behaviour.

1. Whatever player 'B' does, player 'A' is better off confessing the crime.

2. Whatever player 'A' does, player 'B' is better off confessing the crime.

Thus, 'to confess' is the unique Nash equilibrium for this game. But, this is also a dominant strategy for both. Here, each player has the same optimal choice independent of the choice of the other player, which is the dominant strategy equilibrium.

Nash Equilibrium: ***Optimal choice of 'A' given the choice of 'B'.***

Dominant strategy equilibrium: ***Optimal choice of 'A' whatever choice 'B' makes.***

(Confess, Confess) is Pareto inefficient

(Doesn't Confess, Doesn't Confess) is Pareto efficient.

Pareto efficiency condition is the situation in which there is no other option available to make someone better off without making others worse off.

The 'Prisoner's Dilemma'

• **Dominant Strategy:** is the optimal choice of strategy for each player no matter what the other player does.

•**Dominant Strategy of 'A':** 'A' confesses the crime whatever strategy 'B' adopts or makes. This is the strategy that gives him best payoff whatever be the strategy chosen by 'B'.

If he confesses, he gets either 10 or 0 years of imprisonment. If he doesn't confess, he gets either 20 or 5 years of imprisonment. Definitely, 10 is preferred to 20 and 0 is preferred to 5, if the payoff is given in terms of jail term (imprisonment).

•**Dominant Strategy of 'B':** 'B' confesses the crime whatever strategy 'A' adopts or makes. Definitely, confession is the best strategy for 'B' no matter 'A' confesses the crime or not.

If he confesses, he gets either 10 or 0 years of imprisonment. If he doesn't confess, he gets either 20 or 5 years of imprisonment. Definitely, 10 is preferred to 20 and 0 is preferred to 5, if the payoff is given in terms of jail term (imprisonment).

The ‘Prisoner’s Dilemma’

What happens if there doesn’t exist a dominant strategy?

Each player plays **‘Maximin Strategy’**.

Maximin Strategy: It is the strategy chosen by a player to maximise the minimum gain that it can earn.

One who plays the maximin strategy assumes that the opposition will play the strategy that does the most damage.

Repeated Games

In case of 'Prisoner's Dilemma', players met once and played the game single time.

Repeated game is the game which is played repeatedly by the same players. In such cases, the situation is different in that there are new strategic possibilities open to each player.

In a repeated game, each player gets the opportunity to cooperate and thereby to encourage the other player to do the same.

Whether this kind of strategy will be viable or not, it depends on whether the game is going to be played a fixed number of times or an infinite number of times.

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Repeated Games:
Game is played a fixed number of times

- If a game is played a fixed number of times the outcome will be similar to the dominant strategy equilibrium. After all, playing a game for the last time is just like playing it once. So we should expect the same outcome.
- Thus, one can reason that if the players do not cooperate in the last round, they must not have cooperated in all the previous rounds.
- Players cooperate to induce further cooperation. Thus, this will happen if there is a possibility of future play. Since there is no possibility of future play in the last round no one will cooperate then.
- The case is different when the game is played indefinite number of times.

Repeated Games:
Game is played a indefinte number of times

- When the game is played indefinite number of times, each player gets the opportunity of influencing (induces) the opponent's behaviour. If one refuses to cooperate this time, another refuses to cooperate the next time.
- ***As long as both the players care enough about future payoffs, the threat of non-cooperation in the future may be sufficient to convince them to play 'Pareto efficient' strategy.***
- In case of repeated games, 'tit-for-tat' strategy is the simplest strategy. If the opponent cooperated in the previous round, the player would cooperate. If the opponent defected in the previous rounds the player would defect.
- This strategy simply means that 'do whatever the opponent did in the previous round'. It appears to be a good mechanism for achieving the efficient outcome in a 'Prisoner's Dilemma' that will be played an indefinite number of times.

Enforcing a Cartel

- A tit-for-tat strategy

Sequential Games

- The games played either one time or repeated number of times are simultaneous games as both players act simultaneously. But, in many situations it is observed that one player gets to move first and the other player responds. The best example is the ‘Stackelberg’s Model’, where one player acts as the ‘Leader’ and the other acts as the ‘Follower’.

Sequential Games

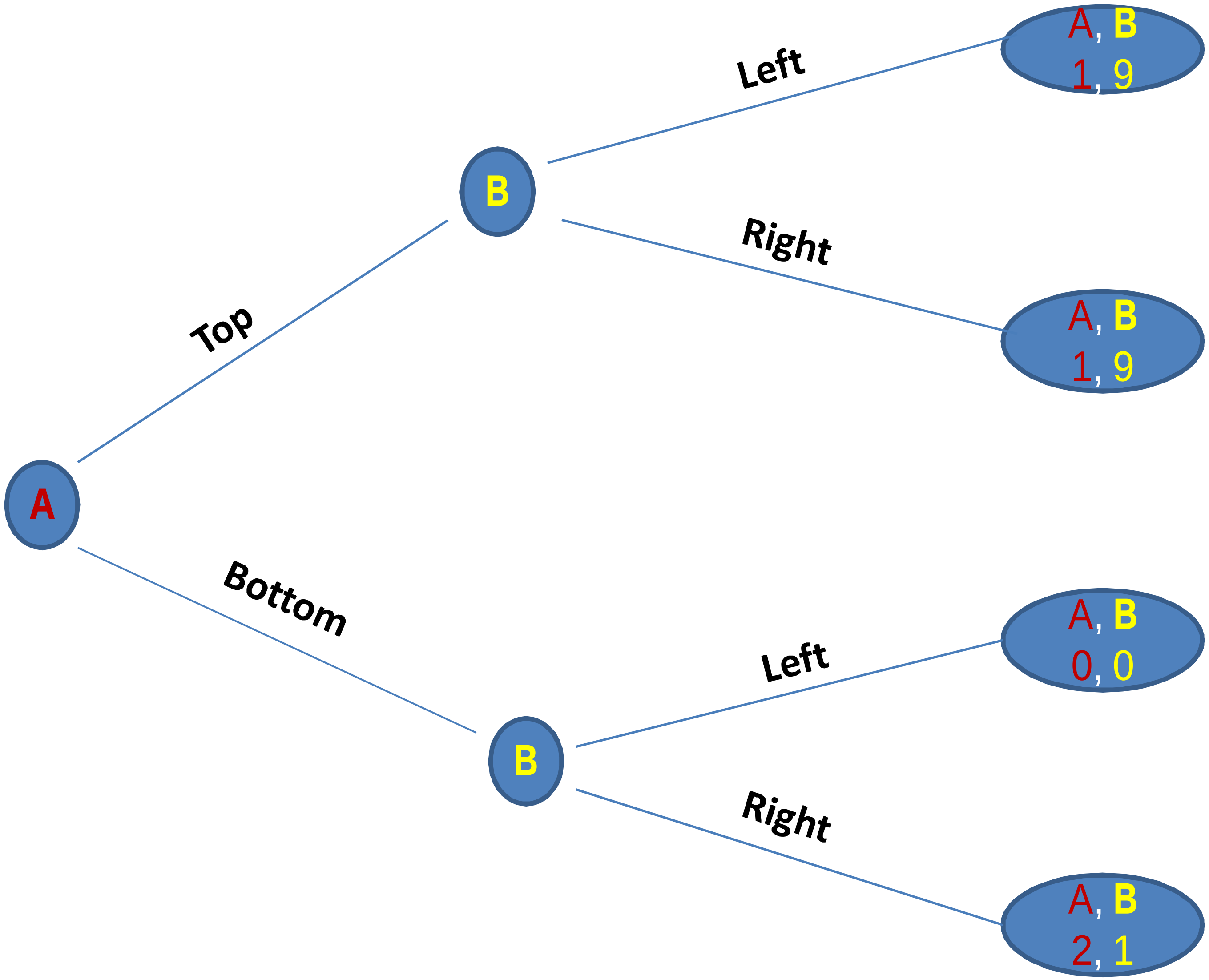
Case of TWO Nash equilibria, but one of them is not reasonable (Stackelberg's Duopoly Model)

		Player B	
		Left	Right
Player A	Top	1, 9	(1,9)
	Bottom	(0,0)	2,1

Sequential Games

- Round 1: Player A chooses top or bottom. Player B observes A’s choice and then chooses either left or right.
- In this form of game, there are two Nash equilibria: (top, left) and (bottom, right). But, one of these equilibria is not really reasonable (Why?).
- The payoff matrix hides the fact that one player gets to know what the other player has chosen before he makes his choice (assymetric nature of the game).

Sequential Games: The order in which players move



Sequential Games

- The diagram illustrates the asymmetric nature of the game, where time pattern of the choices is represented.
- The way to analyse this game is to go to the end and work backward.
- Player A plays top, no matter what B does and the payoff is (1,9).
- Player B plays bottom, the sensible thing for B to do is to choose right and the payoff is (2,1).
- Think about A's initial choice: If he plays top, he gets the payoff 1 and if he plays bottom his payoff is 2. So he will choose bottom. Hence, equilibrium is (bottom, right). The strategies (top, left) are not reasonable equilibrium in this sequential game.

Sequential Games

- From B’s point of view this is rather unfortunate, since he ends up with a payoff of 1 rather than 9! *What might he do about it?*
- Well, he can threaten to play left if A plays bottom.
- Player B’s problem is that once A has made his choice, player A expects player B to do the rational thing. Player B would be better off if he committed himself to play left if player A played bottom.
- One way for B to make such commitment is to hire a lawyer and instruct him to play left if A plays bottom. Then, A will play top. In this case B has done better for himself by ***limiting his choices***.

A Game of Entry Deterrence

- Possibility of entry
- Rational thing for the incumbent to do is to live or let live.